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Real-Time Identification of Flutter Boundaries Using the Discrete Wavelet Transform

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Real-time analysis of an airframe's flutter boundaries during flight testing can help ensure safety and reduce costs. One method of identification is to perform correlation filtering using a set of singlet functions. The method is able to identify accurately the frequency and damping coefficient of the system to excitation, but the computational time required can be too significant to implement in real-time. An alternative method is presented for correlation filtering that employs a multiple-level discrete wavelet transform. The wavelet transform decomposes the response signal into a set of subsignals that correspond to different frequency bands. The same operation is applied to each entry in a dictionary of singlet functions. The transform results in a considerable reduction in the data and, thus, to a reduction in the computational time needed to calculate the correlation. We demonstrate that our approach is able to identify accurately frequency and damping characteristics of the impulse response of both a synthetically generated test signal and actual flight-test data. As a result, real-time identification of flutter boundaries during flight testing may be possible with relatively low-cost computational resources.

Nomenclature

A	=	arbitrary scaling factor
DiH	=	ith subdictionary of high-frequency component
DiL	=	ith subdictionary of low-frequency component
$f(t)$	=	test signal
$g[n]$	=	high-pass filter
$h[n]$	=	low-pass filter
$n(t)$	=	unity-bounded random noise
R	=	set of real numbers
R_+	=	set of positive real numbers
T	=	set of time translation indexes
t	=	time
$x[n]$	=	signal example
$x'_{\text{high}}[k]$	=	detail component of signal
$x'_{\text{low}}[k]$	=	approximation component of signal
Z	=	set of dampings
γ	=	parameter vector
Δt	=	time support range
$\Delta\tau$	=	time period between two consecutive $\tau \cdot \omega$ frequencies
ζ	=	viscous damping ratio
ζ_0	=	damping of signal
ζ	=	damping associated with $\kappa(\tau)$
κ_γ	=	matrix of correlation coefficients
$\kappa(\tau)$	=	highest correlation coefficient during the τth time period
τ	=	time translation index
Ψ	=	singlet function
Ψ_D or $D0$	=	dictionary of singlet functions
$\psi_\gamma(t)$	=	singlet function in dictionary
Ω	=	set of frequencies
ω_0	=	frequency of signal
$\bar{\omega}$	=	frequency associated with $\kappa(\tau)$

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Introduction

MODAL analysis is an important tool for a wide variety of engineering applications including manufacturing,[1] automotive,[2] and aerospace.[3] The information obtained from this analysis describes the natural frequencies and damping associated with rigid-body and structural modes of a system. The information can then be used to indicate properties of the system such as safe operating condition,[4] dynamic response behavior,[5] and material damage.[6]

Flight flutter testing is an application that is dependent on in-flight modal analysis.[7] Essentially, the testing expands the flight envelope to include new operating conditions in an effort to ensure that no aeroelastic instabilities are encountered. Modal analysis is used to indicate the properties of structural modes at test points throughout the flight envelope. The onset of instabilities is identified by noting adverse trends on the modal parameters as the envelope is expanded. Thus, the quality and efficiency of modal analysis is critical for flight flutter testing.[8]

Freudinger et al. developed a method of modal analysis for flutter identification that involves time-domain correlation filtering.[5] They created a dictionary of functions that have similar properties as the impulse response of a single-mode linear system.[9] (Freudinger et al.[5] called their dictionary entries Laplace wavelets; however, we will refer to them as singlet functions to avoid any confusion with the wavelet transform that we apply to the entries in our method.) The modal properties of the system are identified by noting the frequency and damping characteristics of those singlet functions that are highly correlated with the response signal. This method was shown to identify accurately parameters of modal dynamics for several aeroelastic systems[5]; however, the computational requirements of this approach prohibit a cost-effective real-time implementation.

In this paper we present an alternative method for correlation filtering that extends the original continuous-domain concept of Freudinger et al.[5] by employing wavelets. Wavelets have recently been introduced in the context of flight flutter testing.[9,10] Wavelets represent a type of processing that relaxes several constraints on a measurement signal that are assumed to be satisfied when using traditional Fourier processing (see Ref. 11). Wavelet processing has been used extensively for a variety of signal and image processing applications[12–15]; however, they are also receiving attention for analysis of dynamic systems.[5,16]

Our method uses the discrete wavelet transform (DWT) to decompose the response signal into its low- and high-frequency components. Only the low-frequency components that contain information about modal dynamics are retained, and the signal is downsized to reduce the number of points in its representation. The reduced signal can then be further reduced by consecutively applying the discrete wavelet transform or it can be reconstructed to obtain the original time-domain signal. An identical multiresolution decomposition is performed on the dictionary entries to generate a set of multiresolution singlet functions. Correlation filtering is performed using the reduced signal and dictionary entries.

Singlet Functions

In this section we introduce the singlet functions that are used to build the dictionary. The derivation appears elsewhere,[5,17] but we include it here for the convenience of the readers. The singlet function Ψ is a complex, analytic, single-sided damped exponential defined by

$$\psi(\omega, \zeta, \tau, t) = \psi_\gamma(t)$$

$$= \begin{cases} A \exp\left[\left(-\zeta\big/\sqrt{1-\zeta^2}\right)\omega(t-\tau)\right] & \\ \quad \times \exp[-j\omega(t-\tau)] & t \in [\tau, \tau + \Delta t] \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The parameter vector is $\gamma = \{\omega, \zeta, \tau\}$. The elements of γ are related to modal dynamic properties and are frequency $\omega \in R_+$, viscous damping ratio $\zeta \in [0, 1] \subset R_+$, and time translation index $\tau \in R$. The coefficient A is an arbitrary scaling factor that is used to scale each singlet function to a unity norm. The range Δt ensures that the singlet function is compactly supported and has nonzero finite length, but the parameter Δt is generally not explicitly expressed.

Dictionary of the Singlet Functions

A dictionary is a set of singlet functions used for signal decomposition.[11] The dictionary approximates a basis assuming the responses to be analyzed are similar in nature to the singlet functions. The dictionary is defined by the following set of parameters:

$$\Omega = \{\omega_1, \omega_2, \ldots, \omega_p\} \subset R_+$$

$$Z = \{\zeta_1, \zeta_2, \ldots, \zeta_q\} \subset R_+ \cap [0, 1]$$

$$T = \{\tau_1, \tau_2, \ldots, \tau_r\} \subset R \quad (2)$$

The dictionary Ψ_D is defined for the set of entries whose parameters are denoted as follows:

$$\Psi_D = \{\psi_\gamma(\omega, \zeta, \tau, t) : \omega \in \Omega, \zeta \in Z, \tau \in T\} \quad (3)$$

Correlation Filtering

The operation, $\langle\psi_\gamma(t), f(t)\rangle = f(t)^*\psi_\gamma(t)$, correlates a signal with a singlet function in the dictionary. The operation produces a measure of similarity between frequency and damping properties of the dictionary entry ψ_γ and the system that generated the signal $f(t)$. A correlation coefficient $\kappa_\gamma \in R$ is defined to quantify the degree of correlation between each entry and the response signal,

$$\kappa_\gamma = \sqrt{2}\frac{|\langle\psi_\gamma, f(t)\rangle|}{\|\psi_\gamma\|_2\|f\|_2} \quad (4)$$

where κ_γ is a matrix whose dimensions are determined by the parameter vectors of $\{\omega, \zeta, \tau\}$. For online modal analysis, $\kappa(\tau)$ is defined as the peak correlation coefficient at τ between the dictionary entries and the response signal contained in a window that begins at $t = \tau$ and ends at $t = \tau + \Delta t$. We also define $\bar{\omega}$ and $\bar{\zeta}$ as the parameters of the singlet function associated with the peak correlation

$$\kappa(\tau) = \max_{\substack{\omega\in\Omega \\ \zeta\in Z}} \kappa_\gamma = \kappa_{\{\bar{\omega}, \bar{\zeta}, \tau\}} \quad (5)$$

The damping $\bar{\zeta}$ and frequency $\bar{\omega}$ indicate the modal properties of the system that generated the data. A large dictionary of singlet functions must be used to ensure that the dictionary contains a reasonable approximation for the response data, otherwise the correlation filtering will not relate accurate information about the important dynamics. Thus, the computational cost of the correlation filter with inner products can be quite high and can prohibit a cost-effective real-time implementation of this approach.

DWT

We present an approach to correlation filtering using a multiresolution decomposition of the response signal and the entries in the dictionary. We apply the DWT to both the response signal and the singlet functions in the dictionary and compute the correlation level between their subsignal components.

The wavelet transform[18] is a method for complete time–frequency localization for signal analysis and characterization. The wavelet transform of a signal is its decomposition on a family of real orthonormal bases $\psi_{mn}(x)$ obtained through translation and dilation of a kernel function $\psi(x)$ known as the mother wavelet,

$$\psi_{mn}(x) = 2^{-m/2}\psi(2^{-m}x - n) \quad (6)$$

where $m, n \in Z$ are a set of integers. When the orthonormal property of the basis functions is used, wavelet expansion coefficients of a signal $f(x)$ can be computed as

$$c_{mn}(x) = \int_{-\infty}^{\infty} \psi_{m,n}(x) f(x) \quad (7)$$

The signal can be reconstructed from the coefficients as

$$f(x) = \sum_m \sum_n c_{m,n}\psi_{m,n}(x) \quad (8)$$

In general a mother wavelet can be constructed using a scaling function $\phi(x)$ that satisfies the scaling equation

$$\phi(x) = \sum_k h(k)\phi(2x - k) \quad (9)$$

and the corresponding wavelet defining equation

$$\psi(x) = \sum_k g(k)\phi(2x - k) \quad (10)$$

where $g(k) = (-1)^{1-k}h(1-k)$. The coefficients of the scaling equation $h(k)$ must satisfy several conditions for the set of basis functions to be unique, to be orthonormal, and to have a certain degree of regularity. For the filtering operations, $h(k)$ and $g(k)$ coefficients can be used as the impulse responses correspond to the low- and high-pass operations.

As an $\mathcal{O}(N)$ algorithm, the discrete wavelet transform of a given sampled signal is a fast algorithm for computing the wavelet expansion coefficients of the signal. The DWT does not work over all possible scales and locations, but by choosing the scales and locations of the wavelet based on the powers of 2, the accuracy of the algorithm can still be maintained while the computational time is significantly reduced.[19]

Mallat algorithms are used to compute the discrete wavelet transform of a sampled signal.[20] Mallat showed that, using the scaling function ϕ and the wavelet function ψ, the wavelet multiscale representation of the sampled signal can be expressed as[20]

$$f(x) = \sum_k a_{0k}\phi_{0k}(x) + \sum_k \sum_j b_{jk}\psi_{jk}(x) \quad (11)$$

Equation (11) provides a multiscale representation of the signal at the coarse scales $\{0, 1, \ldots, j, \ldots\}$. The Mallat algorithm consists of mapping the coefficients at the single scale to the multiscale coefficients. This transform consists of convolution with a series of filter banks that define the scaling and wavelet functions and down sampling, as shown in Fig. 1.

Correlation Filtering with DWT

Proposed Algorithm

As introduced earlier, we first create a dictionary of time-domain singlet functions. We use the family of signals defined in Eq. (1) because of its similarity to the impulse response of the single-mode system. The DWT is applied to each entry in the dictionary to obtain

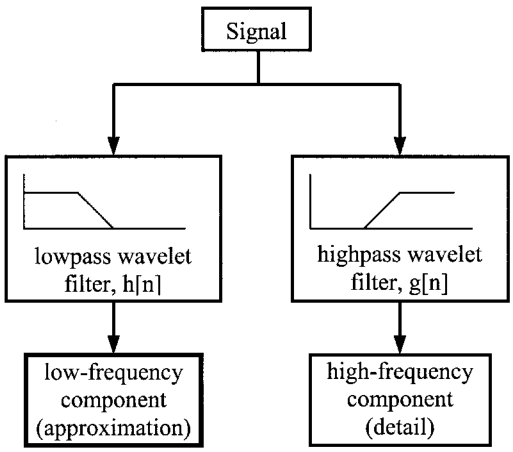

Fig. 1 Decomposition of original signal into low-frequency (approximation) and high-frequency (detail) component using wavelet transforms, $h[n]$ and $g[n]$, respectively.

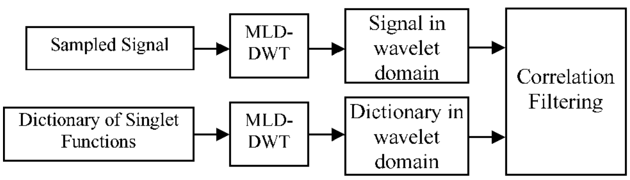

Fig. 2 Method of correlation filtering.

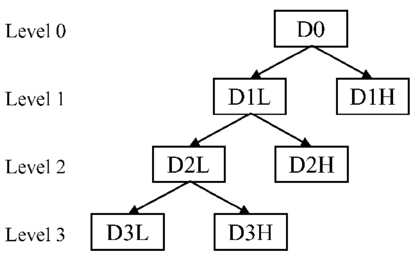

Fig. 3 Three-level multilevel decomposition tree using the wavelet transform.

a wavelet-domain subdictionary (Fig. 2). Multiple levels of decompositions produce a new wavelet-domain dictionary in which the size of each of its entries is significantly reduced compared to the entries in the original dictionary. Similarly, the DWT is applied to the time-domain response signal to obtain the wavelet-domain subsignal.

As shown in Fig. 2, before filtering, a multilevel decomposition using the DWT is applied to both the sampled signal and the dictionary of singlet functions. Correlation filtering results in a match between the singlet function that is the closest approximation of the frequency and damping characteristics of the signal at a specific point in time.

Correlation filtering using a wavelet dictionary consists of, at each τ of the time-domain signal, the correlation coefficient between the subsignal and each entry in the subdictionary is computed. Peaks of the coefficients for a given τ indicate the singlet function with the strongest correlation to the signal at $t = \tau$. During the DWT, the biorthogonal wavelet filter is used as the low-pass and high-pass filter because of its properties of symmetry and compact support.

Multi-Resolution Decomposition Tree

In this section we show how to form the multiresolution decomposition tree of the signal dictionary using the discrete wavelet transform. The original dictionary can be viewed as the level-0 dictionary $D0$ in the multiresolution decomposition tree of the discrete wavelet transform as shown in Fig. 3. We apply the wavelet transform to the level-0 dictionary and create the subdictionaries $D1L$ and $D1H$, where L and H indicate the low- and high-frequency components of the transformed data, respectively. For this study, we only utilize the low-frequency component of each transform because the high-frequency components lacked discriminatory information for our specific task. Subdictionary $D2L$ and $D2H$ are obtained by performing the discrete wavelet transform on $D1L$. The discrete wavelet transform can be further applied to the low-frequency component of each subdictionary entry to decompose further the dictionary and to reduce the number of the points that represent each entry. In this way, we create a multiresolution decomposition tree that includes the subdictionaries $D1L$, $D2L$, and $D3L$. All of the subdictionaries can be obtained offline once the parameter space has been discretized.

When different scales for the wavelets are chosen, the wavelet transform can achieve any desired resolution in time or frequency. Each application of the DWT, increases the frequency resolution of the signal. Thus, the subcomponents at node N of the DWT decomposition tree have a finer frequency resolution than the subcomponents at the node $N - 1$, one level above N. Finer frequency resolution implies better frequency localization allowing us to locate accurately the most prominent frequencies in the original signal. The ability of wavelet transforms to operate locally, that is, extract frequency characteristics in window of a time-based signal, is the significant difference between wavelet transforms and Fourier transforms.

The DWT is also applied to the sampled signal received from the vehicle under test. A multiresolution decomposition tree is generated to obtain the subsignal in each level of resolution. The method is the same as that applied to the entries in the dictionary and its subdictionary already described.

Time-based correlation is computationally demanding. Thus, instead of computing the correlation between the sampled signal and each entry in the original dictionary in the time domain, our method computes the correlation coefficients between the subsignal and each entry of the subdictionary in the lowest level of the decomposition tree.

Experimental Results

Application 1: Test Signal

In our first application, we apply our approach by analyzing the frequency and damping characteristics of a synthetically generated test signal. Our goal is to show that our approach is as capable of identifying the modal parameters as the original method of Freidinger et al.,[5] but with greatly reduced computational requirements.

The test signal $f(t)$, shown in Fig. 4, is used by Freidinger et al.[5] and defined as

$$f(t) = \begin{cases} \exp\left[\left(-\xi_0 \big/ \sqrt{1 - \zeta_0^2}\right)\omega_0(t - t_0)\right] & \\ \quad \times \sin[\omega_0(t - t_0)] + 0.01 \cdot n(t) & t \geq t_0 \\ 0.01 \cdot n(t) & t < t_0 \end{cases} \tag{12}$$

where $\zeta_0 = 0.04$, $\omega_0 = 10$ Hz, and $n(t)$ is a unity-bounded random noise weighted by 0.01. The sampling rate of signal is 200 Hz. The impulse is applied at $t_0 = 0$.

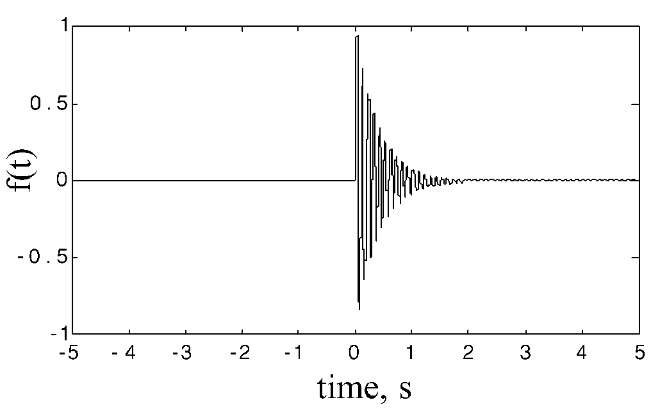

Fig. 4 Test signal (from Freudinger et al.[5]).

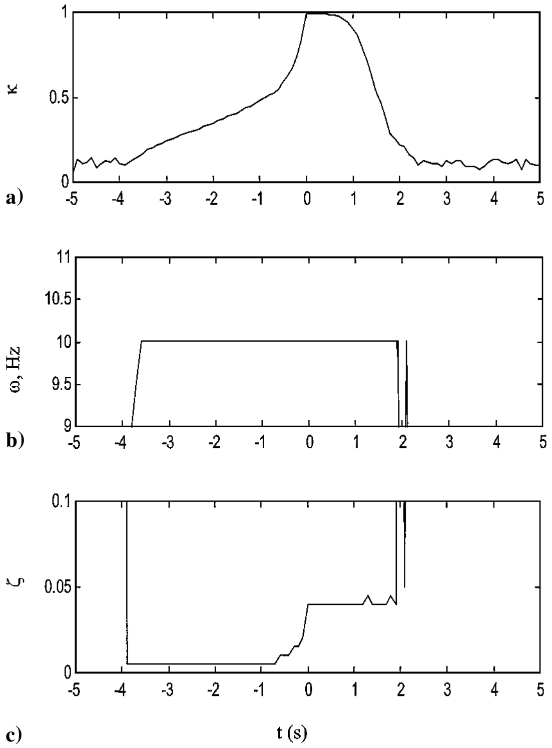

Fig. 5 Results of correlation filtering of the test signal in Fig. 4 and without wavelet decomposition: a) highest correlation coefficient among all singlet functions, b) frequency coefficient, and c) damping coefficient plot of the singlet function with the highest correlation coefficient at that point in time.

We also use the dictionary defined by Freidinger et al.[5] The sets of the parameters are (expressed in the format of {initial value, increment, final value}) as

$$\Omega = \{5:0.5:20\}, \qquad Z = \{\{0.005:0.005:0.2\}\{0.3:0.1:0.9\}\}$$

$$T = \{-5:0.1:5\}$$

The support range of the correlation filtering of the test signal $f(t)$ with the waveforms in the dictionary is $\Delta t = 4$ s. A biorthogonal wavelet filter is used to perform a four-level decomposition of the test signal and dictionary entries. For our purposes, only the approximation component (low-frequency component) is retained at each level for further decomposition.

Results

Figure 5 presents the information obtained by correlation filtering the original (level 0) test signal $f(t)$ with the original dictionary Ψ_D. This method is equivalent to that of Freudinger et al.[5] At the beginning of the impulse, the values of $\kappa(\tau)$ remain close to 1 for about 0.7 s. This demonstrates that there is a high correlation between the signal and some entries in the dictionary. Figures 5b and 5c shows the parameters of $\bar{\omega}$ and $\bar{\zeta}$ of the entries that were strongly correlated with the test signal at each τ obtained from the correlation filtering.

Figure 6a presents the correlation coefficients $\kappa(\tau)$ for each $f(t)$ obtained from the correlation filtering between the low-frequency components of the test signal and the dictionary after applying a three-level DWT. Figures 6b and 6c show the parameters associated with the strongest correlated entries. We can see from Fig. 6 that our method correctly identifies the frequency of the test signal as 10 Hz and settles on the correct damping of $\bar{\zeta} = 0.04$. The results indicate that the fundamental frequency (10 Hz) is included by the output of the low-pass filter at level 3 of the decomposition tree. Additionally at level 3 of the decomposition tree, there are approximately one-eighth the number of samples compared to the original signal. Thus, we greatly reduce the computational requirements of correlation filtering while maintaining the required accuracy. Levels 1 and 2 of the decomposition tree also include the prominent frequency component, and so correlation filtering at those levels will also succeed. However, because they contain more samples, their computational requirements are higher than those at level 3.

Table 1 Average computation time vs DWT decomposition level (ms)

Level	Test signal	DAST signal
0	138.9	126.6
1	74.8	99.1
2	42.8	54.1
3	28.1	32.4

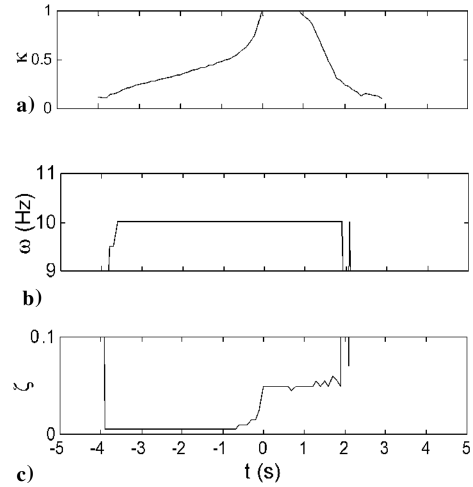

Fig. 6 Results of correlation filtering of the test signal using a three-level discrete wavelet decomposition.

We implemented our algorithm in the C programming language on a 266-MHz personal computer running Windows NT and with 128 MB RAM. Table 1 shows a comparison of average time required to perform correlation filtering as a function of the level of discrete wavelet transform decomposition. A level 0 decomposition is equivalent to performing correlation filtering on the untransformed signal and dictionary entries. The sampling rate of the Drones for Aerodynamic and Structural Testing (DAST) program data requires that, to accomplish a real-time flutter boundary identification, correlation filtering must be performed within 100 ms. The original method can not meet this requirement; however, all levels of wavelet decomposition can be implemented in real time. Filtering with the original signal and dictionary at each τ required about 139 ms. After applying levels 1-3 DWT, the average times for correlation filtering at each level were 74.8, 42.8, and 28.1 ms, respectively. For these cases, the dictionary was transformed in advance and is not included in the recorded time, whereas the signal was transformed in real time and is included in the recorded time. Transformation of the signal in real time does not result in an excessive computational burden. We estimate that about 0.57 ms of time is required for a three-level DWT. A hardware implementation would be considerable faster.

Application 2: Flight-Test Data

In this application, we applied our proposed method on actual flight-test data from NASA's DAST program. The flight data from the DAST aircraft flight is shown in Fig. 7. The data is accelerometer data measured in gravitational acceleration. There is a series of impulse responses in these data that are caused by a pulse to the ailerons approximately every 5 s. The vehicle is increasing speed throughout the data set, becomes unstable at $t = 0$ s, and finally, at $t = 2.5$ s, the vehicle is lost. The sampling rate of the data is 500 Hz.

The dictionary used in this application has 1780 dictionary entries defined as follows in the format of {initial value, increment, final value}:

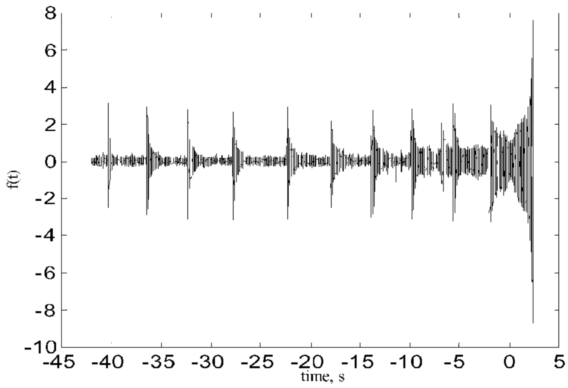

Fig. 7 Flight-test data collected from a NASA DAST.

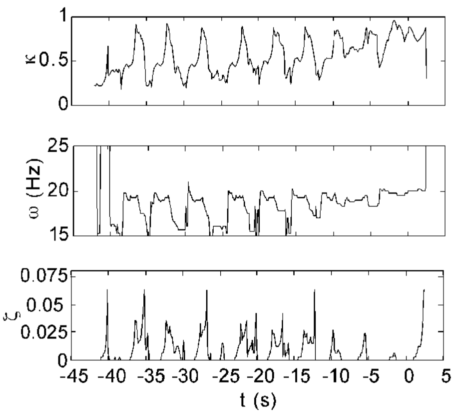

Fig. 8 Results of correlation filtering of the DAST signal without using wavelet decomposition; time required to perform correlation filtering of every element in the signlet dictionary with the signal at time *t* is 126 ms.

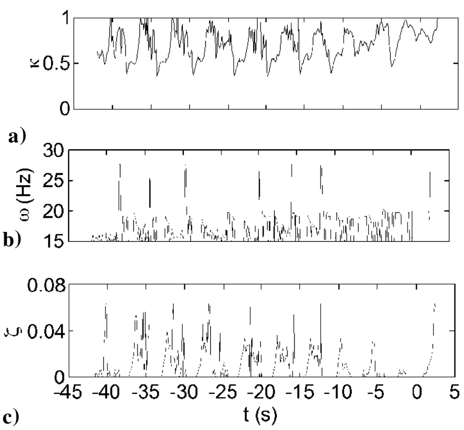

Fig. 9 Results of correlation filtering of the DAST signal with a three-level wavelet decomposition; time required to perform correlation filtering of every element in the singlet dictionary with the signal at time *t* is reduced to 32.4 ms.

$$\Omega = \{10:0.25:30\}, \qquad Z = \{0:0.003:0.063\}$$

$$T = \{-42:0.1:2.5\}$$

The support range is $\Delta t = 2$ s.

In the following results, we employ the same biorthogonal wavelet filter used in the test case to implement a four-level DWT decomposition. Only the approximation component of the signal is used in decomposition. We also implement our method using a Daubechies wavelet filter to test the effects of different wavelet filters.

Results

Figure 8 presents the results of correlation filtering of the original flight data (level 0) with the original dictionary. They are obtained from using the Freudinger et al. method.[5] Figure 9a presents the correlation coefficient $\kappa(\tau)$ between the level-3 decomposition components of the flight data and the dictionary. Figures 9b and 9c present the parameters of $\bar{\omega}$ and $\bar{\zeta}$ obtained from the correlation filtering. At this level, the signal and dictionary entries still retain enough information for us to identify correctly the signal properties of interest.

We also recorded the time required to perform the correlation filtering for each case as we did for the test signal. We used the same 266-MHz personal computer with 128-MB RAM, and the algorithm was implemented in the C programming language. For this algorithm to operate in real time, the computation time for correlation filtering at each τ must be less than $\Delta\tau$, which is the time period between two consecutive correlation operations. In both applications presented here, $\Delta\tau$ is 0.1 s. As shown in the Table 1 the average computation time for the original method (without DWT) is approximately 126.57 ms, which exceeds $\Delta\tau$. As a result, the original method cannot be used in real time (without dedicated hardware or faster computers). However, using a multresolution approach, we find the system modal dynamics in the average time of 99.1, 54.1, and 32.4 ms for a level-1, level-2, and level-3 decomposition, respectively. Because these times are less than $\Delta\tau$, it is possible to perform these calculations in real time, even with the low-cost, general-purpose system we employed.

Conclusions

We have presented a method for real-time identification of flutter boundaries. The method using the discrete wavelet transform to reduce significantly the computational time required for correlation filtering. As a result, low-cost real-time implementation is feasible.

For the application presented here, a traditional approach to downsampling, for example, eliminating every second data point, would also be successful in identifying the flutter boundaries and reducing the computational time required. However, such an approach will result in lost information. The DWT results in lossless data reduction, thus allowing for the signal to be accurately reconstructed, if necessary. Furthermore, although low-frequency information was important in the applications presented here, the DWT allows for the identification of higher-frequency signal components as well. This is an important point considering that the DAST flight data is sampled at 500 Hz and contains frequency components significantly higher than those important for flutter testing.

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